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FINAL REPORT

"Fundamental Investigations in Methods of Operations Research"

U.S. Army Research Office - Durham  
ARO-D Project Number P-9239-M  
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M.I.T. DSR 72650

Contract Period: July 1, 1970 - June 30, 1973

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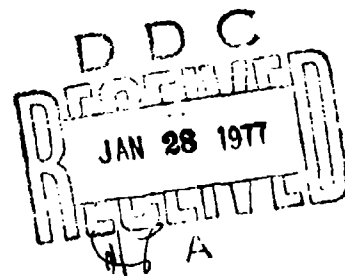
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Contract DAHC04-73-C-0032  
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Contract Period: July 1, 1973 - May 31, 1976

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Professor of Operations  
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## FINAL REPORT TO ARO-D

This final report covers the contract period, July 1, 1970 through May 30, 1976. Professor John D.C. Little, former Director of the Operations Research Center, was Principal Investigator from July 1, 1970 through May 30, 1976. Joining him as Co-Principal Investigator during that period was Professor Jeremy F. Shapiro, former Associate Director of the Operations Research Center. Professor Little was on-leave from M.I.T. from June 1, 1975 through May 30, 1976. At the end of that period he resigned as Director of the Operations Research Center. Professor Shapiro served as Acting Director of the Center during Professor Little's leave; he continues to hold that position.

### STATEMENT OF PROBLEM STUDIED

This project was concerned with basic research into methods of operations research, especially mathematical programming and their applications to a variety of decision problems. Basic research in mathematical programming included the development and testing of a new duality theory for integer and combinatorial programming, new methods for linear fractional programming, a synthesis of duality theory and dynamic programming and results illuminating the relationship between generalized linear programming and mathematical programming duality theory. Additional basic research was performed on Markovian decision models and fixed point theory, network optimization models and multi-attributed utility theory. Mathematical programming methods and models were applied to a variety of problems including machine replacement, energy planning, pollution control and location problems.

PUBLICATIONS supported entirely or in part by ARO-D during the contract period:

#### Journal Articles and Books

Fisher, M.L., W.D. Northup and J.F. Shapiro, "Using Duality to Solve Discrete Optimization Problems: Theory and Computational Experience," Mathematical Programming Study 3, North-Holland Publishing Company, 1975, pp. 56-94.

Gartner, N.H. and J.D.C. Little, "Generalized Combination Method for Area Traffic Control," Transportation Research Record 531, Transportation Research Board, Washington, D.C., 1975.

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Publications, cont.

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Magnanti, T.L., "Complementary Bases of a Matroid," Discrete Mathematics 8, June 1974, pp. 355-361.

Magnanti, T.L., "Fenchel and Lagrange Duality are Equivalent," Mathematical Programming 7, October 1974, pp. 253-258.

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Morse, P.M., "Search Theory and Browsing," The Library Quarterly 40, No. 4, October 1970, pp. 391-408.

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Shapiro, J.F., "Multiple Criteria Public Investment Decision Making by Mixed Integer Programming," in Multiple Criteria Decision Making, edited by H. Thiriez and S. Zionts, Springer-Verlag, 1976, pp. 170-182.

Publications, cont.

Welsch, R.E., "A Weak Convergence Theory for Order Statistics from Strong-Mixing Processes," Annals of Mathematical Statistics 42, No. 5, October 1971, pp. 1637-1646.

Welsch, R.E., "Limit Laws for Extreme Order Statistics from Strong-Mixing Processes," Annals of Mathematical Statistics 43, No. 2, April 1972, pp. 439-446.

Technical Reports and Working Papers

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Bell, D.E., "A Utility Theory Approach to Preferences for Money Over Time," Technical Report No. 72, M.I.T. Operations Research Center, June 1972.

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Gartner, N.H., "Area Traffic Control and Network Equilibrium," Working Paper OR 041-75, M.I.T. Operations Research Center, March 1975.

Gartner, N., J.D.C. Little and H. Gabbay, "Optimization of Traffic Signal Settings in Networks by Mixed-Integer Linear Programming," Technical Report No. 91, M.I.T. Operations Research Center, March 1974.

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Gorry, G.A., W.D. Northup, J.F. Shapiro and C.H. Zakovi, "A Group Theoretic Integer Programming Algorithm: System Design and Computational Experience," Working Paper 513-71, M.I.T. Sloan School of Management, February 1971.

Green, C. and T.L. Magnanti, "Some Abstract Pivot Algorithms," Working Paper OR 031-74, M.I.T. Operations Research Center, February 1974.

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Haehling von Lanzanauer, C. and D.D. Wright, "Developing an Optimal Repair-Replacement Strategy for Pallets," Working Paper OR 047-76, M.I.T. Operations Research Center, February 1976.

Handler, G.Y., "Minimax Network Location: Theory and Algorithms," Technical Report No. 107, M.I.T. Operations Research Center, November 1974.

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Keeney, R.L., "Risk Independence and Multiattributed Utility Functions," Working Paper OR 001-71, M.I.T. Operations Research Center, June 1971.

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Marsten, R.E. and T.L. Morin, "An Algorithm for Nonlinear Knapsack Problems," Technical Report No. 95, M.I.T. Operations Research Center, May 1974.

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Mehta, C.R., "Multiperiod Adaptive Control of the Wellhead Price of Natural Gas: A Bayesian Decision Theoretic Approach," Technical Report No. 100, M.I.T. Operations Research Center, August 1974.

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Patel, N.R., "A Model for Optimal Dynamic Control of Emissions," Working Paper OR 009-72, M.I.T. Operations Research Center, May 1972.

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Shapiro, J.F., "A New Duality Theory for Mixed Integer Programming," Working Paper OR 033-74, M.I.T. Operations Research Center, April 1974.

Shapiro, J.F., "A Survey of Applications of Integer and Combinatorial Programming in Logistics," Working Paper WP-74-35, International Institute for Applied Systems Analysis, Vienna, Austria, August 1974.

Shapiro, J.F., "A Note on the Primal-Dual and Out-of-Kilter Algorithms for Network Optimization Problems," Working Paper OR 040-75, March 1975.

Shapiro, J.F., "Multiple Criteria Public Investment Decision Making," Working Paper 788-75, M.I.T. Sloan School of Management, May 1975.

Shapiro, J.F., "OR Models for Energy Planning," Working Paper 799-75, M.I.T. Sloan School of Management, July 1975.

Shapiro, J.F., "Steepest Ascent Decomposition Methods for Mathematical Programming/Economic Equilibrium Energy Planning Models," Working Paper OR 046-76, M.I.T. Operations Research Center, February 1976.

Shapiro, J.F., "Sensitivity Analysis in Integer Programming," Working Paper OR 048-76, M.I.T. Operations Research Center, February 1976.

Shwimer, J., "Interaction Between Aggregate and Detailed Scheduling in a Job Shop," Technical Report No. 71, M.I.T. Operations Research Center, May 1972.

Tien, J.M., "Control of a Two Customer Class Interactive-Multi-Facility Queuing System," Technical Report No. 73, M.I.T. Operations Research Center, June 1972.

Ting, H.M., "Aggregation of Attributes for Multiattributed Utility Assessment," Technical Report No. 66, M.I.T. Operations Research Center, August 1971.



Publications, cont.

Wagner, M.H., "Constructive Fixed Point Theory and Duality in Nonlinear Programming," Technical Report No. 67, M.I.T. Operations Research Center, September 1971.

Welsch, R.E., "A Weak Convergence Theorem for Order Statistics from Strong-Mixing Processes," Technical Report No. 57, M.I.T. Operations Research Center, December 1970.

Wilde, N.W.G., "Numerical Analysis and Approximation Methods in Discrete-Time Dynamic Programming," Technical Report No. 64, M.I.T. Operations Research Center, June 1971.

SCIENTIFIC PERSONNEL supported entirely or in part by ARO-D during the contract period:

Professor John D.C. Little

November 1, 1970 - May 31, 1971  
September 1, 1971 - May 31, 1972  
September 1, 1972 - May 31, 1973  
September 1, 1973 - May 31, 1974  
September 1, 1974 - May 31, 1975  
September 1, 1975 - January 15, 1976

Professor Thomas L. Magnanti

June 1, 1972 - June 30, 1972  
January 16, 1973 - May 31, 1973  
September 1, 1973 - May 31, 1974  
September 1, 1974 - May 31, 1975

Professor Philip M. Morse

April 1, 1971 - May 31, 1971  
October 1, 1972 - June 30, 1973  
September 1, 1973 - May 31, 1974  
September 1, 1974 - September 30, 1974

Professor Jeremy F. Shapiro

July 1, 1970 - July 31, 1970  
June 1, 1971 - June 30, 1971  
July 1, 1973 - July 31, 1973  
July 1, 1974 - July 31, 1974  
July 1, 1975 - July 31, 1975

Professor Ronald W. Wolff

October 1, 1971 - May 31, 1972

Steven M. Barta

September 1, 1973 - December 31, 1973  
September 1, 1974 - January 15, 1975

David E. Bell

September 1, 1971 - January 15, 1972  
September 1, 1972 - June 30, 1973

Oded Berman

September 1, 1974 - March 31, 1975

Jeremy A. Bloom

September 1, 1973 - September 30, 1973  
March 1, 1974 - March 31, 1974

Robert E. Brooks

July 1, 1975 - August 15, 1975

Kenneth R. Chelst

March 1, 1973 - April 30, 1973

Scientific Personnel, cont.

Samuel S. Chiu  
September 1, 1975 - January 15, 1976

Joseph S. D'Aversa  
January 16, 1974 - May 31, 1974  
September 1, 1975 - January 16, 1976

Rene Dionne  
September 1, 1970 - May 31, 1971

Dorothy P. Elliott  
September 1, 1975 - April 30, 1976

Susan Finger  
September 1, 1974 - May 31, 1975

Henry Gabbay  
January 1, 1973 - May 31, 1973

Bruce L. Golden  
September 1, 1973 - December 31, 1974  
February 1, 1975 - April 30, 1975

John R. Hauser  
September 1, 1974 - January 15, 1975

John A. Mazzarino  
September 1, 1974 - May 31, 1975

Scott A. Neslin  
December 1, 1974 - January 15, 1975

Nitin R. Patel  
September 1, 1972 - January 15, 1973

Jean-Marc Rousseau  
October 1, 1970 - May 31, 1971  
September 1, 1971 - May 31, 1972

Hubert M. Tavernier  
September 1, 1970 - January 31, 1971

Norman W.G. Wilde  
July 1, 1970 - January 15, 1971

DEGREES AWARDED during contract period, supported (in part) by ARC-D:

Abtahi, Saeed, "Application of Integer Programming to Municipal Bond Bidding," S.M. thesis, supervised by Prof. J.F. Shapiro, September 1973.

Degrees Awarded, cont.

Asser, Sylvain, E.T., "An Algorithm to Solve the  $N^{\text{th}}$  Shortest Path Problem," S.M. in O.R. thesis, supervised by Prof. E.G. Frankel and Prof. T.L. Magnanti, February 1974.

Bell, David E., "A Utility Theory Approach to Preferences for Money Over Time," S.M. in O.R. thesis, supervised by Prof. R.L. Keeney, June 1972.

Bell, David E., "The Resolution of Duality Gaps in Discrete Optimization," PhD in O.R. thesis, supervised by Prof. J.F. Shapiro, September 1973.

Bitran, Gabriel R., "Fractional Programming: Duality, Algorithms, Sensitivity Analysis and Applications," S.M. in O.R. thesis, supervised by Prof. T.L. Magnanti, June 1974.

Bitran, Gabriel R., "Admissible Points and Vector Optimization: A Unified Approach," PhD in O.R. thesis, supervised by Prof. T.L. Magnanti, February 1975.

Brooks, Robert E., "Allocation of Natural Gas in Times of Shortage: A Mathematical Programming Model of the Production, Transmission, and Demand for Natural Gas Under Federal Power Commission Regulation," PhD in O.R. thesis, supervised by Prof. T.L. Magnanti, September 1975.

D'Aversa, Joseph S., "Optimal Equipment Maintenance, Rebuilding and Replacement Policy," S.M. in O.R. thesis, supervised by Prof. J.F. Shapiro, June 1972.

Golden, Bruce L., "A Minimum-Cost Multicommodity Network Flow Problem Concerning Imports and Exports," S.M. in O.R. thesis, supervised by Prof. T.L. Magnanti, February 1974.

Hauser, John R., "A Normative Methodology for Predicting Consumer Response to Design Decisions: Issues, Models, Theory and Use," PhD in O.R. thesis, supervised by Prof. J.D.C. Little, June 1975.

Hien, Nguyen Q., "Multi-Depot Vehicle-Routing Problems," S.M. in Mgt., supervised by Prof. T.L. Magnanti, June 1975.

Patel, Nitin R., "Optimal Control of Sulfur Dioxide Emissions at Power Stations: Models and a Case Study," PhD in O.R. thesis, supervised by Prof. J.D.C. Little, June 1973.

Robinson, William O., "Multivariate Probability Assessment," S.M. in E.E. thesis, supervised by Prof. R.L. Keeney, September 1970.

Rousseau, Jean-Marc, "A Cutting Plane Method for the Fixed Cost Problem," PhD in O.R. thesis, supervised by Prof. J.F. Shapiro, September 1973.

Shwimer, Joel, "Interaction Between Aggregate and Detailed Scheduling in a Job Shop," PhD in Mgt. thesis, supervised by Prof. P.R. Kleindorfer, June 1975.

Degrees Awarded, cont.

Tavernier, Hubert M., "Mathematical Analysis of Infinite Horizon Planning Models," PhD in O.R. thesis, supervised by Prof. J.F. Shapiro, February 1971.

Tien, James M., "Control of a Two Customer Class Interactive-Multi-Facility Queuing System," PhD in O.R. thesis, supervised by Prof. G.P. Wadsworth, June 1972.

Wagner, Michael H., "Constructive Fixed Point Theory and Duality in Nonlinear Programming," PhD in Mgt. thesis, supervised by Prof. J.F. Shapiro, September 1971.

Weymuller, Stanislas B., "Duality Theory and Economic Analysis," S.M. in O.R. thesis, supervised by Prof. J.F. Shapiro, June 1972.

Wilde, Norman W.G., "Numerical Analysis and Approximation Methods in Discrete-Time Dynamic Programming," PhD in Math. thesis, supervised by Prof. J.F. Shapiro, June 1971.

ABSTRACTS OF RESEARCH FINDINGS supported entirely or in part by ARO-D during the contract period and cited in this final report were appended to all progress reports and are not included here.